MEASUREMENTS OF THE NEUTRON FLUX ENABLING THE DETERMINATION OF SOME REACTOR CHARACTERISTICS

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A complex of measurements of the neutron flux in a nuclear reactor, based on which sources of radiation of special form are reconstructed uniquely, is studied.

1. If all characteristics of a reactor are known (the total cross section Σ, the scattering function Σ_s, and the source function F), then the incoming neutron flux density at the surface of the reactor ψ and the initial neutron distribution over space and over velocities φ are usually measured in order to determine the neutron flux density u. The function u is reconstructed uniquely from these measurements. Let us assume that the source function F is also unknown. We make an additional measurement X and try to determine the pair functions u and F from the known functions Σ, Σ_s, ψ, φ and χ, i.e., to determine in addition the radiation sources. The question of how, where, and how much to measure in order to determine the pair u and F from these measurements is the main difficulty in the formulation of the inverse problem.

We proceed to the study of the corresponding mathematical models. Under certain assumptions the neutron flux density u satisfies the following linear multivelocity nonstationary anisotropic transfer equation:

$$u_t(t,\ x,\ v)+v\nabla_x u(t,\ x,\ v)=(Pu)(t,\ x,\ v)+F(t,\ x,\ v),$$
$$(t,\ x,\ v)\in(0,\ T)\times\Omega_1\times\Omega_2, \tag{1}$$
$$(Pu)(t,\ x,\ v)=-\sum(t,\ x,\ v)\,u(t,\ x,\ v)+\int_{\Omega_2}\sum\nolimits_s(t,\ x,\ v,\ v')u(t,\ x,\ v')dv';$$

where T is the measurement time; Ω_1 is the region in which the neutron transfer process occurs; and, Ω_2 is the region where the neutron velocities change. In the case of a one-velocity model, Ω_2 is the unit sphere in R^3.

We write the initial and boundary conditions in the form

$$u(0,\ x,\ v)=\varphi(x,\ v),\quad (x,\ v)\in\Omega_1\times\Omega_2, \tag{2}$$

$$u(t,\ x,\ v)=\Psi(t,\ x,\ v),\quad (t,\ x,\ v)\in(0,\ T)\times\partial\Omega_1\times\Omega_2:(n_x,\ v)<0. \tag{3}$$

The problem (1)-(3) has been studied by many authors. Among a large number of works we call attention to [1-6]. The existence and uniqueness of the solution of this problem are proved in the most diverse classes of functions. The answer to the question of where to place additional sensors in order to measure the neutron flux depends primarily on what we know about the source function to be determined. Let us assume that the function F(t, x, v) = f(t)Q(t, x, v) + R(t, x, v), where the functions Q and R are given, while f is unknown (this distribution occurs, for example in the case when the sources are distributed uniformly over the reactor and emit neutrons isotropically; in this case $Q\equiv1$ and $R\equiv0$). We perform a measurement at a fixed point on the surface of the reactor $(x_0\in\partial\Omega_1)$ of the outgoing flux density of neutrons moving with a fixed velocity v_0, i.e., we give the condition of overdetermination

$$u(t,\ x_0,\ v_0)=\chi(t),\quad t\in(0,\ T). \tag{4}$$

The inverse problem of determining the pair of functions u and f from the conditions (1)-(4) was studied in [1, 2]. Under certain conditions a unique solution of this problem exists and is determined by a constructive method. Let us assume that the source function has the form

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$$F(t, x, v) = f(t, v)Q(t, x, v) + R(t, x, v),$$

where the functions Q and R are given, while the function f must be determined. This case occurs, for example, with a uniform but nonisotropic source distribution inside the reactor. We shall measure the neutron flux at an interior point of the region Ω_1, i.e., we shall place a sensor measuring the neutron flux at a given point inside the reactor at all times $t \in (0, T)$. The condition of overdetermination will assume the following form:

$$u(t, x_0, v) = \chi(t, v), \quad (t, v) \in (0, T) \times \Omega_2. \tag{5}$$

The inverse problem of determining the pair of functions u and f from the conditions (1)-(3) and (5) has precisely one solution under definite restrictions on the input data for the problem [1].

Let $x_0 \in \partial\Omega_1$, , i.e., the measurement is carried out on the surface of the reactor. In this case, the solution of the problem (1)-(3) and (5) is not always unique. The solution will not be unique in all cases when some neutrons have a velocity v_0 oriented, in a certain sense, opposite to the direction of the outer normal at the point x_0, i.e., $(n_{x_0}, v_0) < 0$. To simplify the presentation we shall assume that $R \equiv \Sigma_s \equiv \Sigma \equiv 0,\ Q \equiv 1$. We shall study the continuous nonnegative function g(t, v) equal to zero at t = 0 and outside a quite small neighborhood of the vector v_0; in addition, for some $t_0 > 0$ $g(t_0, v_0) = 1$. We set

$$V(t, x, v) = \begin{cases} \int\limits_0^t g(\tau, v)\,d\tau, & t < \alpha(x, -v), \\ \int\limits_{t-\alpha(x,-v)}^t g(\tau, v)\,d\tau, & t \geqslant \alpha(x, -v), \end{cases}$$

where $\alpha(x, v) = \max\{t : x + vt \in \partial\Omega_1\}$.

We verify directly that the pair of functions V and g satisfies the equation

$$V_t(t, x, v) + v\nabla_x V(t, x, v) = g(t, v), \quad (t, x, v) \in D$$

with homogeneous initial (2) and boundary (3) conditions and a homogeneous condition of overdetermination (5). We note that $V \not\equiv 0,\ g \not\equiv 0$. This pair of functions can be added to any solution u and f of the problem (1)-(3), (5) and a different solution can be obtained for the same problem differing from the starting solution. Thus if the solution of the problem (1)-(3) and (5) exists, then it is not unique.

We note that in the case $(n_{x_0}, v) > 0$ for all $v \in \overline{\Omega}_2$ the solution of the problem (1)-(3) and (5) exists and is unique under certain restrictions on the input data of the problem, which can be proved by following the proof of Theorem 2 from [2].

Thus if the source function depends only on t, then to determine it uniquely it is sufficient to measure the neutron flux on the surface of the reactor. If, however, f = f(t, v), then the neutron flux must be measured inside the reactor.

A complex of measurements enabling the determination of nonuniform sources which are constant in time is indicated in [7, 8].

2. In this section we shall present a scheme for determining the pair of functions (u, f) from the conditions (1)-(3) and (5) with $\psi \equiv 0$ using the theory of semigroups.

Let us assume that the source function can be represented in the form

$$F(t, x, v) = f(t, v)Q(t, x, v),$$

where Q is a given function and the function f is to be determined. We note that the method of semigroups was developed for the transfer equation in [5, 6].

We shall study the differential opqerator

$$Au = -v\nabla_x u.$$

in the space of continuous functions C(D) satisfying the condition (3) with $\psi \equiv 0$. It generates the strongly continuous semigroup of operators T(t), whose properties we shall use to solve the inverse problem. We introduce the operators

$$L_1 u = -\sum u + \int\limits_{\Omega_2} \sum\nolimits_s u\,dv', \quad L_2 f = fQ, \quad L_3 u = u(t, x_0, v).$$

Then in the space C(D) there arises the abstract inverse problem:

$$u_t = Au + L_1u + L_2f, \tag{6}$$

$$u|_{t=0} = \varphi, \tag{7}$$

$$L_3u = \chi. \tag{8}$$

The solution of the problems (6) and (7) is written out in terms of the semigroup of the operator A as follows:

$$u(t) = T(t)\varphi + \int_0^t T(t-s)[L_1(s)u(s) + L_2(s)f(s)]\,ds. \tag{9}$$

Using the additional information (8) we obtain one more equation, applying to (6) the operator L_3:

$$\chi'_t(t) = L_3Au + \tilde{L}_1\chi + L_3L_2f, \tag{10}$$

where the operator $\tilde{L}_1$ operates, just as the operator L_1, at some fixed $x = x_0$.

To calculate Au in (10) we must use the formula (9). However, in this case, (10) transforms into a Volterra equation of the first kind and in order to solve efficiently the system of operator integral equations obtained it must be reduced by differentiation to an equation of the second kind. The use of this standard technique gives rise to a derivative in the equations, and as a result the system of integral equations, consisting of Eq. (9) and the differentiated equation (10), must be expanded by adding to it the equation for u'(t). For this we can use Eq. (6), in which the quantity Au is calculated just as before with the help of (9). The calculations yield the following system of operator integral equations for u(t), f(t) and M(t) = u'(t):

$$\begin{aligned} u(t) &= T(t)\varphi + \int_0^t T(t-s)[L_1(s)u(s) + L_2(s)f(s)]\,ds, \\ f(t) &= \tilde{f}(t) + \int_0^t [K(t, s)u(s) + L(t, s)M(s) + R(t, s)f(s)]\,ds, \\ M(t) &= \tilde{M}(t) + \int_0^t [K_1(t, s)u(s) + L_1(t, s)M(s) + R_1(t, s)f(s)]\,ds. \end{aligned} \tag{11}$$

The continuous operator kernels K, L, R, K_1, L_1, R_1 can be written out in an explicit form, but because of the cumbersomeness of these expressions we omit the long calculations.

It is now easy to prove the existence and uniqueness of the solution of the inverse problem. This problem is actually equivalent to the system (11), but this latter system is a Volterra system and its solution is obtained by the method of successive approximations, with which the n-th approximation $u_n(t)$, $f_n(t)$, $M_n(t)$ is obtained from the preceding approximation using the formulas (11), where $u = u_{n-1}$, $f = f_{n-1}$, $M = M_{n-1}$ must be substituted on the right sides of the equations. The stability of the solution of the system (11) and therefore of the inverse problem, which, substituting the explicit form of the functions $\tilde{f}(t)$ and $\tilde{M}(t)$, have the following form:

$$\|u(t)\|_{C^1[0,T]} \leqslant C(\|\varphi\| + \|A\varphi\| + \|\chi\|_{C^1[0,T]}),$$
$$\|f(t)\| \leqslant C(\|\varphi\| + \|A\varphi\| + \|\chi\|_{C^1[0,T]}),$$

follows in a standard manner from Grunwall's lemma.

NOTATION

Σ, total scattering cross section; Σ_s, scattering function; F, source function; u, neutron flux density; ψ, incoming neutron flux density; and, φ, initial neutron spatial and velocity distribution.

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CALCULATION OF A LAMINAR BOUNDARY LAYER ON A ROTATING POROUS DISK

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Averaging of noninertial terms over the boundary-layer section in the equations of motion is used to study the effect of suction and injection on the hydrodynamic flow near a rotating disk.

Control of a boundary layer by suction or injection of one or the other liquid through a porous disk is a technique which is widely used in technology at present [1, 2].

Solution of the laminar boundary-layer equations with consideration of the effect of flow through the porous surface of the body over which the flow takes place is a complex problem which in most cases is solved numerically [3]. However, in a number of technical applications there is a need for analytical expressions for the hydrodynamic flow profiles and boundary-layer thicknesses [4, 5]. In a number of problems the Slezkin–Targa method has been used for this purpose. This method consists of averaging the nonlinear terms in the equations of motion over the boundary-layer thickness [6, 7].

In the present study a modification of this method will be used to calculate the laminar boundary layer in a viscous incompressible liquid on a rotating porous disk of infinite radius in the presence of uniform suction or injection of a liquid with the same physical properties as the main liquid.

In the notation generally used the equations of the spatial boundary layer on a rotating disk have the form [6]:

$$U\frac{\partial U}{\partial r}+W\frac{\partial U}{\partial z}-\frac{V^2}{r}=-\frac{1}{\rho}\frac{\partial P}{\partial r}+\nu\frac{\partial^2 U}{\partial z^2}, \tag{1}$$

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